Sticky Prices, Monopolistic Competition and Nonclearing Labor Markets: Matching an Intertemporal Model to Time Series Data of the US and Euro-area Countries *

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Abstract

Real Business Cycles are often studied in the context of the general equilibrium framework with competitive markets, flexible wages and prices. This restricts the effective application of intertemporal models to the real world business cycles. Stickiness of wages and prices have been considered in recent monopolistic competition models. In this paper, we go a step further and separate labor supply and labor demand

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and allow for different variants of nonclearing labor markets. Calibration for U.S. and a core Euro-area country (Germany) shows that such model variants will produce a higher volatility in employment and a more realistic correlation between consumption and employment and thus fit the data significantly better than the standard model.

Keywords: Wage stickiness, disequilibrium, RBC models, stochastic dynamic models.

JEL classification: E32, C61.
1 Introduction

The real business cycle (RBC) model has become one of the major approaches in macroeconomics to explain observed economic fluctuations. Despite its rather simple structure, it can explain, at least partially, the volatility of some major macroeconomic variables such as output, consumption and capital stock. However, to explain the actual variation in employment the model generally predicts an excessive smoothness of labor effort in contrast to empirical data. This problem of excessive smoothness in labor effort is well-known in the RBC literature. A recent evaluation of this failure of the RBC model is given in Schmidt-Grohe (2001). There the RBC model is compared to indeterminacy models, as developed by Benhabib and his co-authors. Whereas in RBC models the standard deviation of the labor effort is too low, in indeterminacy models it turns out to be excessively high. Another problem in RBC literature related to this, is that the model implies a excessively high correlation between consumption and employment while empirical data only indicates a week correlation. This problem of excessive correlation has, to our knowledge, not sufficiently been studied in the literature. Lastly, the RBC model predicts a significantly high positive correlation between technology and employment whereas empirical research demonstrates, at least at business cycle frequency, a negative or almost zero correlation.

We want to note that the labor market problems, the lack of variation in the employment and the high correlation between consumption and employment in the standard RBC model, may be related to the specification of the labor market, and therefore we could name it the labor market puzzle. In this paper we are mainly concerned with this puzzle. The technology puzzle, that is, the excessively high correlation between technology and employment in the RBC model, has been studied in Gong and Semmler (2004, ch. 5).

Although in the specification of its model structure, the real business cycle model specifies both sides, the demand and supply, of a market, the moments of the economy are however reflected by the variation on one side of the markets due to its general equilibrium nature for all markets (including output, labor and capital markets). For the labor market, the moments of labor effort result from the decision rule of the representative household to supply labor. The variations in labor and consumption both reflect the moments of the two state variables, capital and technology. It is therefore not surprising why employment is highly correlated with consumption and why the variation of consumption is as smooth as labor effort. This further suggests that to resolve the labor market puzzle in a real business cycle model, one has to make improvement upon labor market specifications. One possible approach for such improvement is to introduce the Keynesian feature into the
model and to allow for wage stickiness and a nonclearing labor market.

The research along the line of micro-founded Keynesian economics has been historically developed by the two approaches: one is the disequilibrium analysis, which had been popular before 1980’s and the other is the New Keynesian analysis based on monopolistic competition. Attempts have now been made recently that introduce the Keynesian features into a dynamic optimization model. Rotemberg and Woodford (1995, 1999), King and Wollman (1999), Gali (1999) and Woodford (2003) present a variety of models with monopolistic competition and sticky prices. On the other hand, there are models of efficiency wages where nonclearing labor market could occur.\(^1\) We shall remark that in those studies with nonclearing labor market, an explicit labor demand function is introduced from the perspective of the decision problem of the firm side. However, the decision rule with regard to labor supply in these models is often dropped because the labor supply no longer appears in the utility function of the household. Consequently, the moments of labor effort become purely demand-determined.\(^2\)

In this paper, we will present a stochastic dynamic optimization model including Keynesian features along the line of above consideration. In particular, we shall allow for wage stickiness\(^3\) and nonclearing labor market. However, unlike other recent models that drop the decision rule of labor supply, we view the decision rule of the labor effort as being derived from a dynamic optimization problem as a quite natural way to determine desired labor supply.\(^4\) With the determination of labor demand, derived from the marginal product of labor and other factors,\(^5\) the two basic forces in the labor market

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\(^2\)The labor supply in these models is implicitly assumed to be given exogenously, and normalized to 1. Hence nonclearing of the labor market occurs if the demand is not equal to 1.

\(^3\)Already Keynes (1936) had not only observed a wide-spread phenomenon of downward rigidity of wages but has also attributed strong stabilizing properties of wage stickiness.

\(^4\)One could perceive a change in secular forces concerning labor supply from the side of households, for example, changes in preferences, demographic changes, productivity and real wage, union bargaining, evolution of wealth, taxes and subsides which all affect labor supply. Some of those secular forces are often mentioned in the work by Phelps, see Phelps (1997) and Phelps and Zoega (1998). Recently, concerning Europe, generous unemployment compensation and related welfare state benefits have been added to the list of factors affecting the supply of labor, intensity of job search and unemployment. For an extensive reference to those factors, see Blanchard and Wolfers (2000) and Ljungqvist and Sargent (1998, 2003).

\(^5\)On the demand side one could add beside the pure technology shocks and the real wage, the role of aggregate demand, high interest rates (Phelps 1997, Phelps and Zoega 1998), hiring and firing cost, capital shortages and slow down of growth, for example, in
can be formalized. One of the advantages of this formulation, as will become clear, is that a variety of employment rules could be adopted to specify the realization of actual employment when a nonclearing market emerges. We will assess this model by employing U.S. and German macroeconomic time series data.

Yet before we formally present the model and its calibration we want to note that there is a similarity of our approach chosen here and the New Keynesian analysis. New Keynesian literature presents models with imperfect competition and sluggish price and wage adjustments where labor effort is endogenized. Important work of this type can be found in Rotemberg and Woodford (1995, 1999), King and Wollman (1999), Gali (1999), Erceg, Henderson and Levin (2000) and Woodford (2003). However, the market in those models are still assumed to be cleared since the producer supplies the output according to what the market demands at the existing price. A similar consideration is also assumed to hold for the labor market. Here the wage rate is set optimally by a representative of the household according to the expected market demand curve for labor. Once the wage has been set, it is assumed to be sticky for some time period and only a fraction of wages are set optimally in each period. In those models there will be a gap again between the optimal wage and existing wage, yet the labor market is still cleared since the household is assumed to supply labor whatever the market demand is at the given wage rate.

Below, we shall present a dynamic model that allows for a noncleared labor market, which could be seen to be caused by a staggered wage contract as described by Taylor (1980), Calvo (1983) or other theories of sluggish wage adjustment. The objective to construct a model such as ours is to

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Europe. See Malinvaud (1994) for a more extensive list of those factors.

Another line of recent research on modeling unemployment in a dynamic optimization framework can be found in the work by Merz (1999) who employs search and matching theory to model the labor market, see also Ljungqvist and Sargent (1998, 2003). Yet, unemployment resulting from search and matching problems can rather be viewed as frictional unemployment (see Malinvaud (1994) for his classification of unemployment). As will become clear, this will be different from the unemployment that we will discuss in this chapter.

See, for example, Woodford (2003, ch. 3). There are also traditional Keynesian models that allow for disequilibria, see Benassy (1984) among others. Yet, the well-known problem of these earlier disequilibrium models was that they disregard intertemporal optimizing behavior and never specify who sets the price. This has now been resolved by the modern literature of monopolistic competition as can be found in Woodford (2003). However, while resolving the price setting problem, the decision with regard to quantities seems to be unresolved. The supplier may no longer behave optimally concerning their supply decision, but simply supplies whatever the quantity the market demands for at the current price.

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approach the two aforementioned labor market problems coherently within a single model of dynamic optimization. Yet, we wish to argue that the New Keynesian and our approach are complementary rather than exclusive, and therefore they can somewhat be consolidated as a more complete system for price and quantity determination within the Keynesian tradition. In the current paper we are only concerned with a nonclearing of the labor market as brought into the academic discussion by the disequilibrium school. We will derive the nonclearing of the labor market from optimizing behavior of economic agents but it will be a multiple stage decision process that will generate the nonclearing of the labor market.\footnote{For models with multiple steps of optimization in the context of learning models, see Dawid and Day (2003), Sargent (1998) and Zhang and Semmler (2003).}

The remainder of this paper is organized as follows. Section 2 presents the model structure. Section 3 estimates and calibrates our different model variants for the U.S. economy. Section 4 undertakes the same exercise for the German economy. Section 5 concludes. Appendices I and II contain some technical derivation of the adaptive optimization procedure whereas Appendix III undertakes a welfare comparison of the different model variants.

2 An Economy with Nonclearing Labor Market

We shall still follow the usual assumptions of identical households and identical firms. Therefore we are considering an economy that has two representative agents: the representative household and the representative firm. There are three markets in which the agents exchange their products, labor and capital. The household owns all the factors of production and therefore sells factor services to the firm. The revenue from selling factor services can only be used to buy the goods produced by the firm either for consuming or for accumulating capital. The representative firm owns nothing. It simply hires capital and labor to produce output, sells the output and transfers the profit back to the household.

Unlike the typical RBC model, in which one could assume an once-for-all market, we, however, in this model shall assume that the market to be reopened at the beginning of each period $t$. This is necessary for a model with nonclearing markets in which adjustments should take place which leads us to a multiple stage adaptive optimization behavior. Yet, let us first describe how prices and wages are set.
2.1 The Wage Determination

As usual we presume that both the household and the firm express their desired demand and supply on the basis of given prices, including the output price $p_t$, the wage rate $w_t$ and the rental rate of the capital stock $r_t$, we shall first discuss how the period $t$ prices are determined at the beginning of period $t$. Note that there are three commodities in our model. One of them should serve as a numeraire, which we assume to be the output. Therefore, the output price $p_t$ always equals 1. This indicates that the wage $w_t$ and the rental rate of capital stock $r_t$ are all measured in terms of the physical units of output.\(^9\) As to the rental rate of capital $r_t$, it is assumed to be adjustable so as to clear the capital market. We can then ignore its setting. Indeed, as will become clear, one can imagine any initial value of the rental rate of capital when the firm and the household make the quantity decisions and express their desired demand and supply. This leaves us to focus the discussion only on the wage setting. Let us first discuss how the wage rate might be set.

Most recent literature, in discussing wage setting,\(^{10}\) assumes that it is the supplier of labor, the household or its representative, that sets the wage rate whereas the firm is simply a wage taker. On the other hand, there are also models that discuss how firms set the wage rate.\(^{11}\) In actual bargaining it is likely, as Taylor (1999) has pointed out, that wage setting is an interacting process between firms and households. Despite this variety of wage setting models, we, however, follow the recent approach. We may assume that the wage rate is set by a representative of the household which acts as a monopolistic agent for the supply of labor effort as Woodford (2003, ch. 3) has suggested. Woodford (2003, p.221) introduces different wage setting agents and monopolistic competition since he assumes heterogenous households as different suppliers of differentiated types of labor. In appendix I, in close relationship to Woodford (2003, ch.3), Erceg et al (2000) and Christiano et al. (2001) we present a wage setting model, where wages are set optimally, but a fraction of wages may be sticky. We neglect, however, differentiated types of labor and refer only to aggregate wages.

We want to note, however, that recently many theories have been developed to explain wage and price stickiness. There is the so-called menu

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\(^{9}\)For our simple representative agent model without money, this simplification does not affect our major result derived from our model. Meanwhile, it will allow us to save some effort to explain the nominal price determination, a focus in the recent New Keyensian literature.


\(^{11}\)These are basically the efficiency wage models that are mentioned in the introduction.
cost for changing prices (though this seems more appropriate for the output price). There is also a reputation cost for changing prices and wages.\textsuperscript{12} In addition, changing the price, or wage, needs information, computation and communication, which may be costly.\textsuperscript{13} All these efforts cause costs which may be summarized as adjustment costs of changing the price or wage. The adjustment cost for changing the wage may provide some reason for the representative of the household to stick to the wage rate even if it is known that current wage may not be optimal. One may also derive this stickiness of wages from wage contracts as in Taylor (1980) with the contract period to be longer than one period.

Since workers, or their respective representative, enter usually into long term employment contracts involving labor supply for several periods with a variety of job security arrangements and termination options, a wage may also be understood from an asset price perspective, namely as derivative security based on a fundamental underlying asset such as the asset price of the firm. In principle a wage contract could be treated as a debt contract with similar long term commitment as exists for other liabilities of the firm.\textsuperscript{14} As in the case of the pricing of corporate liabilities the wage contract, the value of the derivative security, would depend on some specifications in contractual agreements. Yet, in general it can be assumed to be arranged for several periods.

As noted above we do not have to posit that the wage rate, $w_t$, to be completely fixed in contracts and not responding to the disequilibrium in the labor market. One may imagine that the dynamics of the wage rate, for example, follows the updating scheme as suggested in Calvo’s staggered price model (1983) or in Taylor’s wage contract model (1980). In Calvo’s model, for example, there is always a fraction of individual prices to be adjusted in each period $t$.\textsuperscript{15} This can be expressed in our model as the expiration of some wage contracts, to be reviewed in each time period and therefore new wage contracts will be signed in each $t$. The new signed wage contracts should respond to the expected market conditions not only in period $t$ but also through $t$ to $t + j$, where $j$ can be regarded as the contract period.\textsuperscript{16} Through such a pattern of wage dynamics, wages are only partially adjusted.

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\textsuperscript{12}This is emphasized by Rotemberg (1982)
\textsuperscript{13}See the discussion in Christiano, Eichenbaum and Evans (2001) and Zbaracki, Ritson, Levy, Dutta and Bergen (2000).
\textsuperscript{14}For such a treatment of the wages as derivative security, see Uhlig (2003). For further details of the pricing of such liabilities, see Grüne and Semmler (2004c).
\textsuperscript{15}These are basically those prices that have not been adjusted for some periods and there the adjustment costs (such as the reputation cost) may not be high.
\textsuperscript{16}This type of wage setting is used in Woodford (2003, ch. 4) and Erceg et al. (2000).
Explicit formulation of wage dynamics of a Calvo type of updating scheme, particularly with differentiated types of labor, is studied in Erceg et al (2000), Christiano et al. (2001) and Woodford (2003, ch. 3) and briefly sketched, as underlying our model, for an aggregate wage in appendix I of this paper. A more explicit treatment is not needed here. Indeed, as will become clear in section 3, the empirical study of our model does not rely on how we formulate the wage dynamics. All we need to presume is that, wage contracts are only partially adjusted, giving rise to a sticky aggregate wage.

2.2 The Household’s Desired Transactions

The next step in our multiple stage decision process is to model the quantity decisions of the households. When the price, including the wage, has been set, the household is then going to express its desire of demand for goods and supply of factors. We define the household’s desired demand and supply as those that can allow the household to obtain the maximum utility on the condition that these demand and supply can be realized at the given set of prices. We can express the household’s desired demand and supply as a sequence of output demand and factor supply \( \{c^d_{t+i}, i^d_{t+i}, n^s_{t+i}, k^s_{t+i+1}\}_{i=0}^\infty \), where \( i_{t+i} \) is referred to investment. Note that here we have used the superscripts \( d \) and \( s \) to refer to the agent’s desired demand and supply. The decision problem for the household to derive its demand and supply can be formulated as

\[
\max_{(c_{t+i}, n_{t+i})_{i=0}^\infty} E_t \left[ \sum_{i=0}^\infty \beta^i U(c^d_{t+i}, n^s_{t+i}) \right] \tag{1}
\]

subject to

\[
c^d_{t+i} + i^d_{t+i} = r_{t+i}k^s_{t+i} + w_{t+i}n^s_{t+i} + \pi_{t+i} \tag{2}
\]

\[
k^s_{t+i+1} = (1 - \delta)k^s_{t+i} + i^d_{t+i} \tag{3}
\]

Above \( \pi_{t+i} \) is the expected dividend. Note that (2) can be regarded as a budget constraint. The equality holds due to the assumption \( U_c > 0 \). Next, we shall consider how the representative household calculates \( \pi_{t+i} \).

Assuming that the household knows the production function \( f(\cdot) \) while it expects that all its optimal plans can be fulfilled at the given price sequence \( \{p_{t+i}, w_{t+i}, r_{t+i}\}_{i=0}^\infty \), we thus obtain

\[
\pi_{t+i} = f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - w_{t+i}n^s_{t+i} - r_{t+i}k^s_{t+i} \tag{4}
\]

Explaining \( \pi_{t+i} \) in (2) in terms of (4) and then substituting from (3) to eliminate \( i^d_{t+i} \), we obtain

\[
k^s_{t+i+1} = (1 - \delta)k^s_{t+i} + f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - c^d_{t+i} \tag{5}
\]
For the given technology sequence \( \{A_{t+i}\}_{i=0}^{\infty} \), equations (1) and (5) form a standard intertemporal decision problem. The solution to this problem can be written as:

\[
\begin{align*}
    c_{t+i}^d &= G_c(k_{t+i}^s, A_{t+i}) \\
    n_{t+i}^s &= G_n(k_{t+i}^s, A_{t+i})
\end{align*}
\]

We shall remark that although the solution appears to be a sequence \( \{c_{t+i}^d, n_{t+i}^s\}_{i=0}^{\infty} \) only \( (c_t^d, n_t^s) \) along with \( (i_t^d, k_t^s) \), where \( i_t^d = f(k_t^s, n_t^s, A_t) - c_t^d \) and \( k_t^s = k_t \), are actually carried into the market by the household for exchange due to our assumption of re-opening of the market.

### 2.3 The Firm’s Desired Transactions

As in the case of the household, the firm’s desired demand for factors and supply of goods are those that maximize the firm’s profit under the condition that all its intentions can be carried out at the given set of prices. The optimization problem for the firm can thus be expressed as being to choose the input demands and output supply \( (n_t^d, k_t^d, y_t^s) \) that maximizes the current profit:

\[
\max y_t^s - r_t k_t^d - w_t n_t^d
\]

subject to

\[
y_t^s = f(A_t, k_t^d, n_t^d)
\]

For regular conditions on the production function \( f(\cdot) \), the solution to the above optimization problem should satisfy

\[
\begin{align*}
    r_t &= f_k(k_t^d, n_t^d, A_t) \\
    w_t &= f_n(k_t^d, n_t^d, A_t)
\end{align*}
\]

where \( f_k(\cdot) \) and \( f_n(\cdot) \) are respectively the marginal products of capital and labor. Next we shall consider the transactions in our three markets. Let us first consider the two factor markets.

### 2.4 Transaction in the Factor Market and Actual Employment

We have assumed the rental rate of capital \( r_t \) to be adjustable in each period and thus the capital market is cleared. This indicates that

\[
k_t = k_t^s = k_t^d
\]
As concerning the labor market, there is no reason to believe that firm’s demand for labor, as implicitly expressed in (10) should be equal to the willingness of the household to supply labor as determined in (7) given the way the wage determination is explained in section 2.1. Therefore, we cannot regard the labor market to be cleared. An illustration of this statement, though in a simpler version, is given in Appendix I.\footnote{Strictly speaking, the so-called labor market clearing should be defined as the condition that the firm’s willingness to demand factors is equal to the household’s willingness to supply factors. Such concept has somehow disappeared in the new Keynesian literature in which the household supplies the labor effort according to the market demand and therefore it does not seem to face excess demand or supply. Yet, even in this case, the household’s willingness to supply labor effort is not necessarily equal to its actual supply or the market demand. At some point the marginal disutility of work may be higher than the pre-set wage. This indicates that even if there are no adjustment costs so that the household can adjust the wage rate at every time period \(t\), the disequilibrium in the labor market may still exist. In Appendix I these points are illustrated in a static version of the working of the labor market.}

Given a nonclearing labor market, we shall have to specify what rule should apply regarding the realization of actual employment.

Disequilibrium Rule: When disequilibrium occurs in the labor market either of the following two rules will be applied:

\[
\begin{align*}
n_t &= \min(n_t^d, n_t^s) \quad \text{(11)} \\
n_t &= \omega n_t^d + (1 - \omega)n_t^s \quad \text{(12)}
\end{align*}
\]

where \(\omega \in (0, 1)\).

Above, the first is the famous short-side rule when nonclearing of the market occurs. It has been widely used in the literature on disequilibrium analysis (see, for instance, Benassy 1975, 1984, among others). The second might be called the compromising rule. This rule indicates that when nonclearing of the labor market occurs both firms and workers have to compromise. If there is excess supply, firms will employ more labor than what they wish to employ.\footnote{This could also be realized by firms by demanding the same (or less) hours per worker but employing more workers than being optimal. This case corresponds to what is discussed in the literature as labor hoarding where firms hesitate to fire workers during a recession because it may be hard to find new workers in the next upswing, see Burnside et al. (1993). Note that in this case firms may be off their marginal product curve and thus this might require wage subsidies for firms as has been suggested by Phelps (1997).} On the other hand, when there is excess demand, workers will have to offer more effort than they wish to offer.\footnote{This could be achieved by employing the same number of workers but each worker supplying more hours (varying shift length and overtime work); for a more formal treatment of this point, see Burnside et al. (1993).} Such mutual
compromises may be due to institutional structures and moral standards of the society. Given the rather corporate relationship of labor and firms in Germany, for example, this compromising rule might be considered a reasonable approximation. Such a rule that seems to hold for many other countries was already discussed early in the economic literature, see Meyers (1968) and also Solow (1979).

We want to note that the unemployment we discuss here is certainly different from the frictional unemployment as often discussed in search and matching models. In our representative agent model, the unemployment is mainly due to adaptive optimization of the household given the institutional arrangements of the wage setting (see section 2.1). The cause for frictional unemployment can arise from informational and institutional search and matching frictions where welfare state and labor market institutions may play a role. Yet the frictions in the institutions of the matching process are likely to explain only a certain fraction of observed unemployment.

2.5 Actual Employment and Transaction in the Product Market

After the transactions in these two factor markets have been carried out, the firm will engage in its production activity. The result is the output supply, which, instead of (8), is now given by

\[
y^*_t = f(k_t, n_t, A_t). \tag{13}
\]

Then the transaction needs to be carried out with respect to \(y^*_t\). It is important to note that when the labor market is not cleared, the previous consumption plan as expressed by (6) becomes invalid due to the improper

\[\text{Note that if firms are off their supply schedule and workers off their demand schedule, a proper study would have to compute the firms’ cost increase and profit loss and the workers’ welfare loss. If, however, the marginal cost for firms is rather flat (as empirical literature has argued, see Blanchard and Fischer, 1989) and the marginal disutility is also rather flat the overall loss may not be so high. The departure of the value function – as measuring the welfare of the representative household from the standard case – is studied in Gong and Semmler (2001). Results of this study are reported in Appendix III of this chapter.}

\[\text{For a recent position representing this view, see Ljungqvist and Sargent (1998, 2003). For comments on this view, see Blanchard (2003), see also Walsh (2002) who employs search and matching theory to derive the persistence of real effects resulting from monetary policy shocks.}

\[\text{Already Hicks (1963) has called this frictional unemployment. Recently, one important form of mismatch in the labor market seems to be the mismatch of skills, see Greiner, Rubart and Semmler (2003).}

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budget constraint (2), which further bring the improper transition law of capital (5), for deriving the plan. Therefore, the household will be required to construct a new consumption plan, which should be derived from the following optimization program:

\[
\max \left( c^d_t, n_t \right) U\left( c^d_t, n_t \right) + E_t \left[ \sum_{i=1}^{\infty} \beta^i U\left( c^d_{t+i}, n^s_{t+i} \right) \right]
\]

subject to

\[k^s_{t+1} = (1 - \delta)k_t + f(k_t, n_t, A_t) - c^d_t\]

\[k^s_{t+i+1} = (1 - \delta)k^s_{t+i} + f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - c^d_{t+i}\]

Note that in this optimization program the only decision variable is about \(c^d_t\) and the data includes not only \(A_t\) and \(k_t\) but also \(n_t\), which is given by either (11) or (12). We can write the solution in terms of the following equation (see Appendix II for details):

\[c^d_t = G_{c2}(k_t, A_t, n_t)\]

Given this adjusted consumption plan, the product market should be cleared if the household demands the amount \(f(k_t, n_t, A_t) - c^d_t\) for investment. Therefore, \(c^d_t\) in (17) should also be the realized consumption.

3 Estimation and Calibration for U. S. Economy

This section provides an empirical study, for the U. S. economy, of our model as presented in the last section. However, the model in the last section is only for illustrative purpose. It is not the model that can be tested with empirical data, not only because we do not specify the forms of production function, utility function and the stochastic process of \(A_t\), but also we do not introduce the growth factor into the model. For an empirically testable model, we here still employ the model as formulated by King, Plosser and Rebelo (1988).

3.1 The Empirically Testable Model

Let \(K_t\) denote for capital stock, \(N_t\) for per capita working hours, \(Y_t\) for output and \(C_t\) for consumption. Assume that the capital stock in the economy follow
the transition law:

\[ K_{t+1} = (1 - \delta)K_t + A_tK_t^{1-\alpha}(N_tX_t)^\alpha - C_t, \]  

(18)

where \( \delta \) is the depreciation rate; \( \alpha \) is the share of labor in the production function \( F(\cdot) = A_tK_t^{1-\alpha}(N_tX_t)^\alpha \); \( A_t \) is the temporary shock in technology and \( X_t \) the permanent shock that follows a growth rate \( \gamma \).\(^{23}\) The model is nonstationary due to \( X_t \). To transform the model into a stationary setting, we divide both sides of equation (18) by \( X_t \):

\[ k_{t+1} = \frac{1}{1 + \gamma} \left[ (1 - \delta)k_t + A_tk_t^{1-\alpha}(n_t\hat{N}/0.3)^\alpha - c_t \right], \]  

(19)

where \( k_t \equiv K_t/X_t, c_t \equiv C_t/X_t \) and \( n_t \equiv 0.3N_t/\hat{N} \) with \( \hat{N} \) to be the sample mean of \( N_t \). Note that \( n_t \) is often regarded to be the normalized hours. The sample mean of \( n_t \) is equal to 30 \%, which, as pointed out by Hansen (1985), is the average percentage of hours attributed to work. Note that the above formulation also indicates that the form of \( f(\cdot) \) in the previous section may follow

\[ f(\cdot) = A_tk_t^{1-\alpha}(n_t\hat{N}/0.3)^\alpha \]  

(20)

while \( y_t \equiv Y_t/X_t \) with \( Y_t \) to be the empirical output.

With regard to the household preference, we shall assume that the utility function takes the form

\[ U(c_t, n_t) = \log c_t + \theta \log(1 - n_t) \]  

(21)

The temporary shock \( A_t \) may follow an AR(1) process:

\[ A_{t+1} = a_0 + a_1A_t + \epsilon_{t+1}, \]  

(22)

where \( \epsilon_t \) is an independently and identically distributed (\textit{i.i.d.}) innovation: \( \epsilon_t \sim N(0, \sigma^2) \).

### 3.2 The Data Generating Process

For our empirical test, we consider three model variants: the standard RBC model, as a benchmark for comparison, and the two labor market disequilibrium models with the disequilibrium rules as expressed in (11) and (12) respectively. Specifically, we shall call the standard model the Model I; the disequilibrium model with short side rule (11) the Model II; and the disequilibrium model with the compromising rule (12) the Model III.

\(^{23}\)Note that \( X_t \) includes both population and productivity growth.
For the standard RBC model, the data generating process include (19), (22) as well as

\[ c_t = G_{11}A_t + G_{12}k_t + g_1 \]  (23)
\[ n_t = G_{21}A_t + G_{22}k_t + g_2 \]  (24)

Note that here (23) and (24) are the linear approximations to (6) and (7) when we ignore the superscripts \( s \) and \( d \). The coefficients \( G_{ij} \) and \( g_i \)(\( i = 1, 2 \) and \( j = 1, 2 \)) are the complicated functions of the model’s structural parameters, \( \alpha, \beta \), among others. They are computed as in Gong and Semmler (2004, ch. 5) by the numerical algorithm using the linear-quadratic approximation method presented in Gong and Semmler (2004, chs. 1-2). Given these coefficients and the parameters in equation (22), including \( \sigma_\varepsilon \), we can simulate the model to generate stochastically simulated data. These data can then be compared to the sample moments of the observed economy.

Obviously, the standard model does not allow for nonclearing of the labor market. The moments of the labor effort are solely reflected by the decision rule (24) which is quite similar in its structure to the other decision rule given by (23), i.e., they are both determined by \( k_t \) and \( A_t \). This structural similarity are expected to produce two labor market puzzles as aforementioned:

- First, the volatility of the labor effort can not be much different from the volatility of consumption, which generally appears to be smooth.

- Second, the moments of labor effort and consumption are likely to be strongly correlated.

To define the data generating process for our disequilibrium models, we shall first modify (24) as

\[ n_t^* = G_{21}A_t + G_{22}k_t + g_2 \]  (25)

On the other hand, the equilibrium in the product market indicates that \( c_t^d \) in (17) should be equal to \( c_t \). Therefore, this equation can also be approximated as

\[ c_t = G_{31}A_t + G_{32}k_t + G_{33}n_t + g_3 \]  (26)

In the appendix, we provide the details how to compute the coefficients \( G_{3j} \), \( j = 1, 2, 3 \), and \( g_3 \).

Next we consider the labor demand derived from the production function \( F(\cdot) = A_tK_t^{1-\alpha}(N_tX_t)^\alpha \). Let \( X_t = Z_tL_t \), with \( Z_t \) to be the permanent shock resulting purely from productivity growth, and \( L_t \) from population growth. We shall assume that \( L_t \) has a constant growth rate \( \mu \) and hence \( Z_t \) follows

15
the growth rate ($\gamma - \mu$). The production function can be written as $Y_t = A_t Z_t^\alpha K_t^{1-\alpha} H_t^\beta$, where $H_t$ equals $N_t L_t$, which can be regarded as total labor hours. Taking the partial derivative with respect to $H_t$ and recognizing that the marginal product of labor is equal to the real wage, we thus obtain

$$w_t = \alpha A_t Z_t k_t^{1-\alpha} (n_t^d \bar{N}/0.3)^{\alpha-1}$$

This equation is equivalent to (10). It generates the demand for labor as

$$n_t^d = (\alpha A_t Z_t/w_t)^{1/(1-\alpha)} k_t (0.3/\bar{N}). \quad (27)$$

Note that the per capita hours demanded $n_t^d$ should be stationary if the real wage $w_t$ and productivity $Z_t$ grow at the same rate. This seems to be roughly consistent with the U.S. experience that we shall now calibrate.

Thus, for the nonclearing market model with short side rule, Model II, the data generating process includes (19), (22), (11), (25), (26) and (27) with $w_t$ given by the observed wage rate. We thereby do not attempt to give the actually observed sequence of wages a further theoretical foundation.\textsuperscript{24} For our purpose it suffices to take the empirically observed series of wages. For Model III, we use (12) instead of (11).

### 3.3 The Data and the Parameters

Before we calibrate the models we shall first specify the parameters. There are altogether 10 parameters in our three variants: $a_0$, $a_1$, $\sigma_\varepsilon$, $\gamma$, $\mu$, $\alpha$, $\beta$, $\delta$, $\theta$, and $\omega$. We first specify $\alpha$ and $\gamma$ respectively at 0.58 and 0.0045, which are standard. This allows us to compute the data series of the temporary shock $A_t$. With this data series, we estimate the parameters $a_0$, $a_1$ and $\sigma_\varepsilon$. The next three parameters $\beta$, $\delta$ and $\theta$ are estimated with the GMM method by matching the moments of the model generated by (19), (23) and (25). The estimation is conducted by a global optimization algorithm called simulated annealing. These parameters have already been estimated in Gong and Semmler (2004, ch. 5), and therefore we shall employ them here. For the new parameters, we specify $\mu$ at 0.001, which is close to the average growth rate of the labor force in U.S.; the parameter $\omega$ in Model III is set to 0.1203. It is estimated by minimizing the residual sum of square between actual employment and the model generated employment. The estimation is executed by a conventional algorithm, the grid search. Table 1 illustrates these parameters:

\textsuperscript{24}One however might apply here the efficiency wage theory or other theories such as the staggered contract theory that justify the wage stickiness.
Table 1: Parameters Used for Calibration

<table>
<thead>
<tr>
<th></th>
<th>(a_0)</th>
<th>0.0333</th>
<th>(\sigma_\varepsilon)</th>
<th>0.0185</th>
<th>(\mu)</th>
<th>0.0010</th>
<th>(\beta)</th>
<th>0.9930</th>
<th>(\theta)</th>
<th>2.0189</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_1)</td>
<td>0.9811</td>
<td>(\gamma)</td>
<td>0.0045</td>
<td>(\alpha)</td>
<td>0.5800</td>
<td>(\delta)</td>
<td>0.2080</td>
<td>(\omega)</td>
<td>0.1203</td>
</tr>
</tbody>
</table>

The data set used in this section is taken from Christiano (1987). The wage series are obtained from Citibase. It is re-scaled to match the model’s implication.\(^{25}\)

### 3.4 Calibration

Table 2 reports our calibration from 5000 stochastic simulations. The results in this table are confirmed by Figure 1, where a one time simulation with the observed innovation \(A_t\) are presented.\(^{26}\) All time series are detrended by the HP-filter.

\(^{25}\)Note that this re-scaling is necessary because we do not exactly know the initial condition of \(Z_t\), which we set equal to 1. We re-scaled the wage series in such a way that the first observation of employment is equal to the demand for labor as specified by equation (27).

\(^{26}\)In Gong and Semmler (2004, ch. 5) we show that \(A_t\) as can be computed as a Solow residual which may reflect also the demand shock in addition to the technology shock.
Table 2: Calibration of the Model Variants: U.S. Economy (numbers in parentheses are the corresponding standard errors)

<table>
<thead>
<tr>
<th>Standard Deviations</th>
<th>Consumption</th>
<th>Capital</th>
<th>Employment</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Economy</td>
<td>0.0081</td>
<td>0.0035</td>
<td>0.0165</td>
<td>0.0156</td>
</tr>
<tr>
<td>Model I Economy</td>
<td>0.0091</td>
<td>0.0036</td>
<td>0.0051</td>
<td>0.0158</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0007)</td>
<td>(0.0006)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>Model II Economy</td>
<td>0.0137</td>
<td>0.0095</td>
<td>0.0545</td>
<td>0.0393</td>
</tr>
<tr>
<td></td>
<td>(0.0098)</td>
<td>(0.0031)</td>
<td>(0.0198)</td>
<td>(0.0115)</td>
</tr>
<tr>
<td>Model III Economy</td>
<td>0.0066</td>
<td>0.0052</td>
<td>0.0135</td>
<td>0.0197</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0010)</td>
<td>(0.0020)</td>
<td>(0.0026)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Economy</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Capital Stock</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Model I Economy</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Capital Stock</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Model II Economy</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Capital Stock</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Model III Economy</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Capital Stock</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>Output</td>
</tr>
</tbody>
</table>
First we want to remark that the structural parameters that we used here for calibration are estimated by matching the Model I Economy to the Sample Economy. The result, reflected in Table 2, is therefore somewhat biased in favor of the Model I Economy. It is not surprising that for most variables the moments generated from the Model I Economy are closer to the moments of the Sample Economy. Yet even in this case, there is an excessive smoothness of the labor effort and the employment series of the data cannot be matched. For our time period, 1955.1 to 1983.4, we find 0.32 in the Model I Economy as the ratio of the standard deviation of labor effort to the standard deviation of output. This ratio is roughly 1 in the Sample Economy. The problem is, however, resolved in our Model II and Model III Economies representing sticky wages and labor market nonclearing. There the ratio is 1.38 and 0.69 for the Model II and Model III Economies respectively.

Further evidence on the better fit of the nonclearing labor market models – as concerns the volatility of the macroeconomic variables – is also demonstrated in the Figure 1 where the horizontal figures show, from top to bottom, actual (solid line) and simulated data (dotted line) for consumption, capital stock, employment and output, the three columns representing the figures for Model I, Model II and Model III Economies. As observable, in particular the Model III Economy fits, along most dimensions, best the actual data. As can be seen from the separate figures, the volatility of employment has been greatly increased for both Model II and Model III. In particular, the volatility in the Model III Economy is close to the one in the Sample Economy, although too high a volatility is observable in the Model II Economy which may reflect our assumption that there are no search and matching frictions (which, of course, in the actual economy will not hold). We therefore may conclude that Model III is the best in matching the labor market volatility.

We want to note that the failure of the standard model to match the volatility of employment of the data is also described in the recent paper by Schmidt-Grohe (2001). For her employed time series data 1948.3 - 1997.4, Schmidt-Grohe (2001) finds that the ratio of the standard deviation of employment to the standard deviation of output is roughly 0.95, close to our Sample Economy. Yet for the standard RBC model, the ratio is found to be 0.49, which is too low compared to the empirical data. For the indeterminacy model, originating in the work by Benhabib and co-authors, she finds the ratio to be 1.45, which seems too high. As noted above, a similarly high ratio of standard deviations can also be observed in our Model II Economy where the short side rule leads to excessive fluctuations of the labor effort.

Next, let us look at the cross-correlations of the macroeconomic variables. In the Sample Economy, there are two significant correlations we can observe: the correlation between consumption and output, roughly 0.75, and between
employment and output, about 0.72. These two strong correlations can also be found in all of our simulated economies. However, in our Model I Economy and this only holds for the Model I Economy (the standard RBC model) in addition to these two correlations, consumption and employment are, with 0.93, also strongly correlated. Yet, empirically, this correlation is weak, about 0.46.

The latter result of the standard model is not surprising given that movements of employment as well as consumption reflect the movements in the state variables capital stock and the temporary shock. They, therefore, should be somewhat correlated. We remark here that such an excessive correlation has, to our knowledge, not explicitly been discussed in the RBC literature, including the recent study by Schmidt-Grohe (2001). Discussions have often focused on the correlation with output.
A success of our nonclearing labor market models, see the Model II and III Economies, is that employment is no longer significantly correlated with consumption. This is because we have made a distinction between the demand and supply of labor, whereas only the latter, labor supply, reflects the moments of capital and technology as consumption does. Since the realized employment is not necessarily the same as the labor supply, the correlation with consumption is therefore weakened.
4 Estimation and Calibration for the German Economy

Above we have employed a model with nonclearing labor market for the U. S. economy. We have seen that one of the major reasons that the standard model can not appropriately replicate the variation in employment is its lack of introducing the demand for labor. Next, we pursue a similar study of German economy. For this purpose we shall first summarize some stylized facts on the German economy compared to the U.S. economy.

4.1 The Data

Our subsequent study of the German economy employs the time series data from 1960.1 to 1992.1. We thus have included a short period after the unification of Germany (1990 - 1991). We use again quarterly data. The time series data on GDP, consumption, investment and capital stock are OECD data, see OECD (1998a), the data on total labor force is also from the OECD (1998b). The time series data on total working hours is taken from Statistisches Bundesamt (1998). The time series on the hourly real wage index is from OECD (1998a).

4.2 The Stylized Facts

Next, we want to compare some stylized facts. Figures 2 and 3 compare 6 key variables relevant for the models for both the German and U.S. economies. In particular, the data in Figure 3 are detrended by the HP-filter. The standard deviations of the detrended series are summarized in Table 3.
Figure 2: Comparison of Macroeconomic Variables U. S. versus Germany
Figure 3: Comparison of Macroeconomic Variables: U. S. versus Germany (data series are detrended by the HP-filter)
Several remarks are at place here. First, employment and the efficiency wage are among the variables with the highest volatility in the U. S. economy. However, in the German economy they are the smoothest variables. Second, the employment (measured in terms of per capita hours) is declining over time in Germany (see Figure 2 for the non-detrended series), while in the U.S. economy, the series is approximately stationary. Third, in the U. S. economy, the capital stock and temporary shock to technology are both relatively smooth. In contrast, they are both more volatile in Germany. These results might be due to our first remark regarding the difference in employment volatility. The volatility of output must be absorbed by some factors in the production function. If employment is smooth, the other two factors have to be volatile.

Should we expect that such differences will lead to different calibration of our model variants? This will be explored next.

### 4.3 The Parameters

For the German economy, our investigation showed that an AR(1) process does not match well the observed process of $A_t$. Instead, we shall use an AR(2) process:

$$A_{t+1} = a_0 + a_1 A_t + a_2 A_{t-1} + \varepsilon_{t+1}$$

The parameters used for calibration are given in Table 8.4. All of these parameters are estimated in the same way as those for the U.S. economy.
Table 4: Parameters used for Calibration (German Economy)

<table>
<thead>
<tr>
<th>(a_0)</th>
<th>0.0044</th>
<th>(\gamma)</th>
<th>0.0083</th>
<th>(\delta)</th>
<th>0.0538</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>1.8880</td>
<td>(\mu)</td>
<td>0.0019</td>
<td>(\theta)</td>
<td>2.1507</td>
</tr>
<tr>
<td>(a_2)</td>
<td>-0.8920</td>
<td>(\alpha)</td>
<td>0.6600</td>
<td>(\omega)</td>
<td>0</td>
</tr>
<tr>
<td>(\sigma_\epsilon)</td>
<td>0.0071</td>
<td>(\beta)</td>
<td>0.9876</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is important to note that the estimated \(\omega\) in this case is on the boundary 0, indicating the weight of the demand is zero in the compromising rule (12). In other words, the Model III Economy is almost identical to the Model I Economy. This seems to provide us with the conjecture that the Model I Economy, the standard model, will be the best in matching German labor market.

4.4 Calibration

As for the U.S. economy we provide in Table 5 for the German economy the calibration result from 5000 time stochastic simulations. In Figure 4 we again compare the one-time simulation with the observed \(A_t\) for our model variants. Again all time series here are detrended by the HP-filter. ²⁷

²⁷Note that we do not include the Model III Economy for calibration. Due to the zero value of the weighting parameter \(\omega\), the Model III Economy is equivalent to the Model I Economy.
Table 5: Calibration of the Model Variants: German Economy (number in parentheses are the corresponding standard errors)

<table>
<thead>
<tr>
<th></th>
<th>Consumption</th>
<th>Capital</th>
<th>Employment</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Economy</td>
<td>0.0146</td>
<td>0.0203</td>
<td>0.0100</td>
<td>0.0258</td>
</tr>
<tr>
<td>Model I Economy</td>
<td>0.0292</td>
<td>0.0241</td>
<td>0.0107</td>
<td>0.0397</td>
</tr>
<tr>
<td></td>
<td>(0.0106)</td>
<td>(0.0066)</td>
<td>(0.0023)</td>
<td>(0.0112)</td>
</tr>
<tr>
<td>Model II Economy</td>
<td>0.1276</td>
<td>0.0425</td>
<td>0.0865</td>
<td>0.4648</td>
</tr>
<tr>
<td></td>
<td>(0.1533)</td>
<td>(0.0238)</td>
<td>(0.1519)</td>
<td>(0.9002)</td>
</tr>
<tr>
<td><strong>Correlation Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.4360</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>0.0039</td>
<td>-0.3002</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.9692</td>
<td>0.5423</td>
<td>0.0202</td>
<td>1.0000</td>
</tr>
<tr>
<td>Model I Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.7208</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0920)</td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>0.5138</td>
<td>-0.1842</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1640)</td>
<td>(0.1309)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.9473</td>
<td>0.4855</td>
<td>0.7496</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>(0.0200)</td>
<td>(0.1099)</td>
<td>(0.1028)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Model II Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.6907</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1461)</td>
<td>(0.0000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>0.7147</td>
<td>0.3486</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2319)</td>
<td>(0.4561)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.8935</td>
<td>0.5420</td>
<td>0.9130</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>(0.1047)</td>
<td>(0.2362)</td>
<td>(0.1312)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>
In contrast to U.S. economy we find some major differences. First, there is a difference concerning the variation of employment. The standard problem of excessive smoothness with respect to employment in the benchmark model no longer holds for the German economy. This is likely to be due to the fact that employment itself is smooth in the German economy (see Table 3 and Figure 3). We shall also note that the simulated labor supply in Germany is smoother than in the U. S. (see Figure 5). In most labor market studies the German labor market is often considered less flexible than the U. S. labor market. In particular, there are stronger influences of labor unions and various legal restrictions on firms’ hiring and firing decisions.\textsuperscript{28} Such influences and legal restriction will give rise to the smoother employment series in contrast to the U. S. Such influences and legal restriction, or what Solow (1979) has termed the moral factor in the labor market, may also be viewed as a readiness to compromise as our Model III suggests. Those factors

\textsuperscript{28}See, for example, Nickell (1997) and Nickell (2003), and see already Meyers (1964).
will indeed give rise to a smooth employment series.

Further, if we look at the labor demand and supply in Figure 5, the supply of labor is mostly the short side in the Germany economy whereas in U.S. economy demand is dominating in most periods. Note that here we must distinguish the supply that is actually provided in the labor market and the “supply” that is specified by the decision rule in the standard model. It might reasonably be argued that due to the intertemporal optimization subject to the budget constraints the supply specified by the decision rule may only approximate the decisions from those households for which unemployment is not expected to pose a problem on their budgets. Such households are more likely to be currently employed and protected by labor unions and legal restrictions. In other words, currently employed labor decides, through the optimal decision rule, about labor supply and not those who are currently unemployed. Such a shortcoming of a single representative agent intertemporal decision model could presumably be overcome by an intertemporal model with heterogenous households.\textsuperscript{29}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Comparison of demand and supply in the labor market (solid line for actual, dashed line for demand and dotted line for supply)}
\end{figure}

\textsuperscript{29}See, for example, Uhlig and Xu (1996).
The second difference concerns the trend in employment growth and unemployment of the U.S. and Germany. So far we only have shown that our model of nonclearing labor market seems to match better than the standard RBC model the variation in employment. This in particular seems to be true for the U.S. economy. We did not attempt to explain the trend of the unemployment rate neither for the U.S. nor for Germany. We want to note that the time series data (U.S. 1955.1 - 1983.1, Germany 1960.1 - 1992.1) are from a period where the U.S. had higher – but falling – unemployment rates, whereas Germany had still lower but rising unemployment rates. Yet, since the end of the 1980s the level of the unemployment rate in Germany has considerably moved up, partly due to the unification of Germany after 1989.

5 Differences in Labor Market Institutions

In section 2 we have introduced rules that might be thought to be operative when there is a nonclearing labor market. In this respect, as our calibration in section 3 has shown, the most promising route to model, and to match, stylized facts of the labor market, through a microbased labor market behavior, is the compromising model. One hereby may pay attention to some institutional characteristics of the labor market presumed in our model.

The first is the way how the agency representing the household sets the wage rate. If the household sets the wage rate, as if it were monopolistic competitor, then at this wage rate the household’s willingness to supply labor is likely to be less than the market demand for labor unless the household sufficiently under-estimates the market demand when it conducts its optimization for wage setting. Such a way of wage setting may imply unemployment and it is likely to be the institutional structure that gives the representative household (or the representative of the household, such as unions), the power to bargain with the firm in wage setting. Yet, there could be, of course, other reasons why wages do not move to a labor market clearing level – such as efficiency wage, insider – outsider relationship, a wage determined by standards of fairness as in Solow (1979) or by long term contractual agreements (as discussed in section 2.1).

On the other hand, there can be labor market institutions, for example corporatist structures, also measured by our \( \omega \), which affect the actual employment. Our \( \omega \) expresses how much weight is given to the desired labor

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30 This is similar to Woodford’s (2003, ch. 3) idea of a deviation of the efficient and natural level of output where the efficient level is achieved only in a competitive economy with no frictions.
supply or desired labor demand. A small $\omega$ means that the agency, representing the household, has a high weight in determining the outcome of the employment compromise. A high $\omega$ means that the firm’s side is stronger in employment negotiations. As our empirical estimations in Gong, Ernst and Semmler (2004) have shown the former case, a low $\omega$, is very characteristic of Germany, France and Italy whereas a larger $\omega$ is found for U.S. and the U.K.

Given the rather corporatist relationship of labor and the firm in some European countries, with some considerable labor market regulations through legislature and union bargaining (rules of employment protection, hiring and firing restrictions, extension of employment even if there is a shortfall of sales etc.), our $\omega$ may thus measure differences concerning labor market institutions between the U.S. and European countries. This has already been stated in the 1960s by Meyers. He states: “One of the differences between the United States and Europe lies in our attitude toward layoffs... When business falls off, he [the typical American employer] soon begins to think of reduction in work force... In many other industrial countries, specific laws, collective agreements, or vigorous public opinion protect the workers against layoffs except under the most critical circumstances. Despite falling demand, the employer counts on retraining his permanent employees. He is obliged to find work for them to do... These arrangements are certainly effective in holding down unemployment”. (Meyers, 1964:)

Thus, we wish to argue that the major international differences causing employment variations do arise less from real wage stickiness (due to the presence of unions and the extend and duration of contractual agreements between labor and the firm) but rather it seems to be the degree to which compromising rules exist and which side dominates the compromising rule. A lower $\omega$, defining, for example, the compromising rule in Euro-area countries, can show up as difference in the variation of macroeconomic variables. This is demonstrated in section .4 for the German economy.

We there could observe that first, employment and the efficiency wage (defined as real wage scaled down by productivity) are among the variables

\footnote{In the paper by Gong, Ernst and Semmler (2004) it is also shown that the $\omega$ is strongly negatively correlated with labor market institutions.}

\footnote{This could also be realized by firms by demanding the same (or less) hours per worker but employing more workers than being optimal. The case would then correspond to what is discussed in the literature as labor hoarding where firms hesitate to fire workers during a recession because it may be hard to find new workers in the next upswing, see Burnside et al. (1993). Note that in this case firms may be off their marginal product curve and thus this might require wage subsidies for firms as has been suggested by Phelps (1997).}

\footnote{In fact real wage rigidities in the U.S. are almost the same as in European countries, see Flaschel, Gong and Semmler (2001).}
with the highest volatility in the U. S. economy. However, in the German economy they are the smoothest variables. Second, in the U. S. economy, the capital stock and temporary shock to technology are both relatively smooth. In contrast, they are both more volatile in Germany. These results are likely to be due to our first remark regarding the difference in employment volatility. The volatility of output must be absorbed by some factors in the production function. If employment is smooth, the other two factors have to be volatile.

Indeed, recent Phillips curve studies do not seem to reveal much difference in real wage stickiness between Germany and the U.S., although the German labor market is often considered less flexible. Yet, there are differences in another sense. In Germany, there are stronger influences of labor unions and various legal restrictions on firms’ hiring and firing decisions shorter work week even for the same pay etc. Such influences and legal restriction will give rise to the smoother employment series in contrast to the U.S.. Such influences and legal restriction, or what Solow (1979) has termed the moral factor in the labor market, may also be viewed as a readiness to compromise as our Model III suggests. Those factors will indeed give rise to a lower ω and a smoother employment series.

So far we only have shown that our model of nonclearing labor market seems to match better the variation in employment than the standard RBC model. Yet, we did not attempt to explain the secular trend of the unemployment rate neither for the U.S. nor for Germany. We want to express a conjecture of how our model can be used to study the trend shift in employment. We want to note that the time series data for the table 3 (U.S. 1955.1-1983.1, Germany 1960.1-1992.1) are from a period where the U.S. had higher – but falling – unemployment rates, whereas Germany had still lower but rising unemployment rates. Yet, since the end of the 1980s the level of the unemployment rate in Germany has considerably moved up, partly, of course due to the unification of Germany after 1989.

One recent attempt to better fit the RBC model’s predictions with labor

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34 See Flaschel, Gong and Semmler (2001).
35 See, for example, Nickell (1997) and Nickell et al. (2003), and see already Meyers (1964).
36 It might reasonably be argued that, due to intertemporal optimization subject to the budget constraints, the supply specified by the decision rule may only approximate the decisions of those households for which unemployment is not expected to pose a problem on their budgets. Such households are more likely to be currently employed represented by labor unions and covered by legal restrictions. In other words, currently employed labor decides, through the optimal decision rule, about labor supply and not those who are currently unemployed. Such a feature could presumably be better studied by an intertemporal model with heterogenous households, see, for example, Uhlig and Xu (1996).
market data has employed search and matching theory.\textsuperscript{37} Informational or institutional search frictions may then explain the equilibrium unemployment rate and its rise. Yet, those models usually observe that there has been a shift in matching functions due to evolution of unemployment rates such as, for example, experienced in Europe since the 1980s, and that the model itself fails to explain such a shift.\textsuperscript{38}

In contrast to the literature on institutional frictions in the search and matching process we think that the essential impact on the trend in the rate of unemployment seems to stem from both changes of preferences of households as well as a changing trend in the technology shock.\textsuperscript{39} Concerning the latter, as shown in Gong and Semmler (2004, chs. 5 and 9), the Solow residual, as it used in RBC models as the technology shock, greatly depends on endogenous variables (such as capacity utilization). Thus exogenous technology shocks constitute only a small fraction of the Solow residual. We thus might conclude that cyclical fluctuations in output and employment are not likely to sufficiently be explained by productivity shocks alone. Gali (1999) and Francis and Ramey (2001, 2003) have argued that other shocks, for example demand shocks, are important as well.

Yet, in the long run, the change in the trend of the unemployment rate is likely to be related to the long-run trend in the true technology shock. Empirical evidence on the role of lagging implementation and diffusion of new technology for low employment growth in Germany can be found in Heckman (2003) and Greiner, Semmler and Gong (2004). In the context of our model this would have the effect that labor demand, given by equation (27) may fall short of labor supply given by equation (24). This is likely to occur in the long-run if the productivity $Z_t$ in equation (27) starts tending to grow at a lower rate which many researchers recently have maintained to have happened in Germany, and other European countries, since the 1980s.\textsuperscript{40} Yet, as recent research has stressed, for example, the work by Phelps, see Phelps (1997) and Phelps and Zoega (1998), there have also been secular changes on the supply side of labor due to changes in preferences of households.\textsuperscript{41} Some


\textsuperscript{38}For an evaluation of the search and matching theory as well as the role of shocks to explain the evolution of unemployment in Europe, see Blanchard and Wolfers (2000) and Blanchard (2003).

\textsuperscript{39}See Campbell (1994) for a modelling of a trend in technology shocks.

\textsuperscript{40}Of course, the trend in the wage rate is also important in the equation for labor demand (in equation 25). For an account of the technology trend, see Flaschel, Gong and Semmler (2001), and for an additional account of the wage rate, see Heckman (2003).

\textsuperscript{41}Phelps and his co-authors have pointed out that an important change in the households’ preferences in Europe is that households now rely more on assets instead of labor income.
of those factors affecting the households’ supply of labor have been discussed above.

6 Conclusions

Market clearing is a prominent feature in the standard RBC model which commonly presumes wage and price flexibility. In this paper, we have introduced an adaptive optimization behavior and a multiple stage decision process that, given wage stickiness, results in a nonclearing labor market in an otherwise standard stochastic dynamic model. Nonclearing labor market is then a result of different employment rules derived on the basis of a multiple stage decision process. Calibrations have shown that such model variants will produce a higher volatility in employment, and thus fit the data significantly better than the standard model.\footnote{Appendix III computes the welfare loss of our different model variants of nonclearing labor market. There we find that similarly to Sargent and Ljungqvist (1998), that the welfare losses are very small.}

As concerning international aspects of our study we presume that different labor market institutions result in different weights defining the compromising rule. The results for Euro-area economies, for example, for Germany in contrast to the U.S., are consistent with what has been found in many other empirical studies with regard to the institutions of the labor market.

Finally, with respect to the trend of lower employment growth in some European countries as compared to the U.S. since the 1980s, our model suggests that one has to study more carefully the secular forces affecting the supply and the demand of labor as modeled in our multiple stage decision process of section 2. In particular, on the demand side for labor, the slow down of technology seems to have been a major factor for the low employment growth in Germany and other countries in Europe.\footnote{See Blanchard and Wolfers (2000), Greiner, Semmler and Gong (2004) and Heckman (2003)} On the other hand there has also been changes in the preferences of households. Our study has provided a framework that allows to also follow up such issues.\footnote{For further discussion, see also Gong and Semmler (2004, ch. 9).}
7 Appendix I: Wage Setting

Suppose now that at the beginning of \( t \) the household (of course with certain probability denoted as \( 1 - \xi \)) decides to set up a new wage rate \( w_t^* \) given the data \((A_t, k_t)\), and the sequence of expectations on \( \{A_{t+i}\}_{i=1}^{\infty} \) where \( A_t \) and \( k_t \) are referred to as the technology and capital stock respectively. If the household knows the production \( f(A_t, k_t, n_t, \ldots) \), where \( n_t \) the labor effort so that it may also know the firm’s demand for labor, the decision problem of the household with regard to wage setting may be expressed as follows:

\[
\max_{w_t^*, \{c_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} (\xi \beta)^i U(c_{t+i}, n(w_t^*, k_{t+i}, A_{t+i})) \right]
\]

subject to

\[
k_{t+i+1} = (1 - \delta) k_{t+i} + f(A_{t+i}, k_{t+i}, n(w_t^*, k_{t+i}, A_{t+i})) - c_{t+i}
\]

Above \( \xi \) is the probability that the new wage rate \( w_t^* \) will still be effective in period \( t + i \). Obviously, this probability will be reduced when \( i \) become larger. \( U(\cdot) \) is the household’s utility function, which depends on consumption \( c_{t+i} \) and the labor effort \( n(w_t^*, k_{t+i}, A_{t+i}) \). Note that here \( n(w_t^*, k_{t+i}, A_{t+i}) \) is the function of firm’s demand for labor, which is derived from the condition of marginal product equal to the wage rate:

\[
w_t^* = f_n(A_{t+i}, k_{t+i}, n_{t+i})
\]

We shall remark that although the decision is mainly about the choice of \( w_t^* \), the sequence of \( \{c_{t+i}\}_{i=0}^{\infty} \) should also be considered for the dynamic optimization. Of course there is no guarantee that the household will actually implement this sequence \( \{c_{t+i}\}_{i=0}^{\infty} \). However, as argued by recent New Keynesian literature, there is only a certain probability (due to the adjustment cost in changing the wage) that the household will set a new wage rate in period \( t \). Therefore, the observed wage dynamics \( w_t \) may follow Calvo’s updating scheme:

\[
w_t = (1 - \xi) w_t^* + \xi w_{t-1}
\]

Such a wage indicates that there exists a gap between optimum wage \( w_t^* \) and the observed wage \( w_t \).

It should be noted that in recent New Keynesian literature where the wage is set in a similar way as we have discussed here, the concept of non-clearing labor market somehow disappeared. In this literature, the household is assumed to supply the labor effort according to the market demand at the
existing wage rate and therefore does not seem to face the problem of excess demand or supply. Instead, what New Keynesian economists are concerned with is the gap between the optimum price and actual price, whose existence is caused by the adjustment cost in changing prices. In correspondence to the gap between optimum and actual price, there also exists a gap between optimum output and actual output.

Figure 6: A Static Version of the Working of the Labor Market

Some clarifications may be obtained by referring to a static version of our view on the working of the labor market. In figure 6, the supplier (or the household, in the labor market case) first (say, at the beginning of period 0) sets its price optimally according to the expected demand curve $D_0$. Let us denote this price as $w_0$. Consider now the situation that the supplier’s expectation on demand is not fulfilled. Instead of $n_0$, the market demand at $w_0$ is $n'$. In this case, the household may reasonably believe that the demand curve should be $D'$ and therefore the optimum price should be $w^*$ while the optimum supply should be $n^*$. Yet, due to the adjustment cost in changing prices, the supplier may stick to $w_0$. This produces the gaps between optimum price $w^*$ and actual price $w_0$ and between optimum supply $n^*$ and actual supply $n'$.

However, the existence of price and output gaps does not exclude the existence of a disequilibrium or nonclearing market. New Keynesian litera-
ture presumes that at the existing wage rate, the household supplies labor effort whatever the market demand for labor is. Note that in figure 6 the household’s willingness to supply labor is $n_s$. In this context the marginal cost curve, $MC$, can be interpreted as marginal disutility of labor which has also an upward slope since we use the standard log utility function as in the RBC literature. This then means that the household’s supply of labor will be restricted by a wage rate below, or equal, to the marginal disutility of work. If we define the labor market demand and supply in a standard way, that is, at the given wage rate there is a firm’s willingness to demand labor and the household’s willingness to supply labor, and a nonclearing labor market can be very general phenomena. This indicates that even if there are no adjustment costs so that the household can adjust the wage rate in every $t$ (so that there is no price and quantity gaps as we have mentioned earlier), the disequilibrium in the labor market may still exist.

**Appendix II: Adaptive Optimization and Consumption Decision**

For the problem (14) - (16), we define the Lagrangian:

$$L = E_t \left\{ \log c^d_t + \theta \log(1 - n_t) \right\} + \lambda_t \left[ k^s_{t+1} - \frac{1}{1 + \gamma} \left[ (1 - \delta)k^s_t + f(k^s_t, n_t, A_t) - c^d_t \right] \right\} + E_t \left\{ \sum_{i=1}^{\infty} \beta^i \left[ \log(c^d_{t+i}) + \theta \log(1 - n^s_{t+i}) \right] + \beta^i \lambda_{t+i} \left[ k^s_{t+1+i} - \frac{1}{1 + \gamma} \left[ (1 - \delta)k^s_{t+i} + f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - c^d_{t+i} \right] \right] \right\}.$$

Since the decision is only about $c^d_t$, we thus take the partial derivatives of $L$ with respect to $c^d_t$, $k^s_{t+1}$ and $\lambda_t$. This gives us the following first-order condition:

$$\frac{1}{c^d_t} - \frac{\lambda_t}{1 + \gamma} = 0,$$

$$\frac{\beta}{1 + \gamma} E_t \left\{ \lambda_{t+1} \left[ (1 - \delta) + (1 - \alpha)A_{t+1} (k^s_{t+1})^{-\alpha} \left( n^s_{t+1} \bar{N} / 0.3 \right)^\alpha \right] \right\} = \lambda_t, \quad (31)$$

$$k^s_{t+1} = \frac{1}{1 + \gamma} \left[ (1 - \delta)k^s_t + A_t(k^s_t)^{1-\alpha} \left( n_t \bar{N} / 0.3 \right)^\alpha - c^d_t \right]. \quad (32)$$

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Recall that in deriving the decision rules as expressed in (23) and (24) we have postulated

\begin{align}
\lambda_{t+1} &= H k_{t+1}^s + QA_{t+1} + h, \\
n_{t+1}^s &= G_{21} k_{t+1}^s + G_{22} A_{t+1} + g, \tag{33}
\end{align}

where \( H, Q, h, G_{21}, G_{22} \) and \( g \) have all been resolved previously in the household optimization program. We therefore obtain from (33) and (34)

\begin{align}
E_t \lambda_{t+1} &= H k_{t+1}^s + Q(a_0 + a_1 A_t) + h, \tag{35} \\
E_t n_{t+1}^s &= G_{22} k_{t+1}^s + D_2(a_0 + a_1 A_t) + g. \tag{36}
\end{align}

Our next step is to linearize (30) - (32) around the steady states. Suppose they can be written as

\begin{align}
F_{c1} c_t + F_{c2} A_t + f_c &= 0, \tag{37} \\
F_{k1} E_t \lambda_{t+1} + F_{k2} E_t A_{t+1} + F_{k3} k_{t+1}^s + F_{k4} E_t n_{t+1}^s + f_k &= \lambda_t, \tag{38} \\
k_{t+1}^s &= A k_t + W A_t + C_1 c_t^d + C_2 n_t + b. \tag{39}
\end{align}

Expressing \( E_t \lambda_{t+1}, E_t n_{t+1}^s \) and \( E_t A_{t+1} \) in terms of (35), (36) and \( a_0 + a_1 A_t \) respectively, we obtain from (38)

\begin{align}
\kappa_1 k_{t+1}^s + \kappa_2 A_t + \kappa_0 &= \lambda_t, \tag{40}
\end{align}

where, in particular,

\begin{align}
\kappa_0 &= F_{k1} (Qa_0 + h) + F_{k2} a_0 + F_{k4} (G_{22} a_0 + g_2) + f_k, \tag{41} \\
\kappa_1 &= F_{k1} H + F_{k3} + F_{k4} G_{21}, \tag{42} \\
\kappa_2 &= F_{k1} Q a_1 + F_{k2} a_1 + F_{k4} G_{22} a_1. \tag{43}
\end{align}

Using (37) to express \( \lambda_t \) in (40), we further obtain

\begin{align}
\kappa_1 k_{t+1}^s + \kappa_2 A_t + \kappa_0 &= - \frac{F_{c1}}{F_{c2}} c_t^d - \frac{f_c}{F_{c2}}, \tag{44}
\end{align}

which is equivalent to

\begin{align}
k_{t+1}^s &= - \frac{\kappa_2}{\kappa_1} A_t - \frac{F_{c1}}{F_{c2} \kappa_1} c_t^d - \frac{\kappa_0}{\kappa_1} - \frac{f_c}{F_{c2} \kappa_1}. \tag{45}
\end{align}

Comparing the right side of (39) and (45) will allow us to solve \( c_t^d \) as

\[ c_t^d = - \left( \frac{F_{c1}}{F_{c2} \kappa_1} + C_1 \right)^{-1} \left[ A k_t + \left( \frac{\kappa_2}{\kappa_1} + W \right) A_t + C_2 n_t + \left( b + \frac{\kappa_0}{\kappa_1} \right) A_t + \frac{f_c}{F_{c2} \kappa_1} \right]. \]
Appendix III: Welfare Comparison of the Model Variants

In this appendix we want to undertake a welfare comparison of our different model variants. We follow here Ljungqvist and Sargent (1998) and compute the welfare implication of the different model variants. Yet, whereas they concentrate on the steady state, we compute the welfare also outside the steady state. We here restrict our welfare analysis to the U. S. model variants. It is sufficient to consider only the equilibrium (benchmark) model, and the two models with nonclearing labor market. They are given by Simulated Economy I, II, and III. A likely conjecture is that the benchmark model should always be superior to the other two variants because the decisions on labor supply - which are optimal for the representative agent - are realized in all periods.

However, we believe that this may not generically be the case. The point here is that the model specification in variants II and III, is somewhat different from the the benchmark model due to the distinction between expected and actual moments with respect to our state variable, the capital stock. In the models of nonclearing market the representative agent may not rationally expect those moments of the capital stock. The expected moments are represented by equation (5) while the actual moments are expressed by equation (5). They are not necessary equal unless the labor efforts of those two equations are equal. Also, in addition to $A_t$, there is another external variable $w_t$, entering into the models, which will affect the labor employed (via demand for labor) and hence eventually the welfare performance. The welfare result due to these changes in the specification may therefore deviate from what one would expect.

Our exercise here is to compute the values of the objective function for all our three models, given the sequence of our two decision variables, consumption and employment. Note that for our models variants with nonclearing labor market, we use realized employment, rather than the decisions on labor supply, to compute the utility functional. More specifically, we calculate $V$, where

$$V = \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

where $U(c_t, n_t)$ is given by $\log(c_t) + \theta \log(1 - n_t)$. This exercise here is conducted for different initial conditions of $k_t$ denoted by $k_0$. We choose the different $k_0$ based on the grid search around the steady state of $k_t$. Obviously, the value of $V$ for any given $k_0$ will also depend on the external variable $A_t$. 

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and \( w_t \) (though in the benchmark model, only \( A_t \) appears). We consider two different ways to treat these external variables. One is to set both external variables at their steady state levels for all \( t \). The other is to employ their observed series entering into the computation. Figure 7 provides the welfare comparison of the two versions.

In Figure 7, the percentage deviations of \( V \) from the corresponding values of benchmark model is plotted for both Model II and Model III given for various \( k_0 \) around the steady states. The various \( k_0 \)’s are expressed in terms of the percentages deviation from the steady state of \( k_t \).

It is not surprising to find that in most cases the benchmark model is the best in its welfare performance, since most of the values are negative. However, it is important to note that the deviations from the benchmark model...
model are very small. Similar results have been obtained by Ljungqvist and Sargent (1998), they, however, compare only the steady states. Meanwhile, not always is the benchmark model the best one. When $k_0$ is sufficiently high, close to or higher than the steady state of $k_t$, the deviations become 0 for the Model II. Furthermore, in the case of using observed external variables, the Model III will be superior in its welfare performance when $k_0$ is larger than its steady state, see lower part of the figure.
References


