The El Farol Problem Revisited

by

Volker Böhm

Department of Economics
and
Institut for Mathematical Economics (IMW)
Bielefeld University
P.O. Box 100 131
D–33501 Bielefeld, Germany
e-mail: vboehm@wiwi.uni-bielefeld.de
web: www.wiwi.uni-bielefeld.de/boehm/

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Abstract

The so-called El Farol problem describes a prototypical situation of interacting agents making binary choices to participate in a non-cooperative environment or to stay by themselves and choosing an outside option. In a much cited paper Arthur (1994) argues that persistent non-converging sequences of rates of participation with permanent forecasting errors occur due to the non-existence of a prediction model for agents to forecast the attendance appropriately to induce stable rational expectations solutions. From this he concludes the need for agents to use boundedly rational rules.

This note shows that in a large class of such models the failure of agents to find rational prediction rules which stabilize is not due to a non-existence of perfect rules, but rather to the failure of agents to identify the correct class of predictors from which the perfect ones can be chosen. What appears as a need to search for boundedly rational predictors originates from the non-existence of stable confirming self-referential orbits induced by predictors selected from the wrong class.

Specifically, it is shown that, within a specified class of the model and due to a structural non-convexity (or discontinuity), symmetric Nash equilibria of the associated static game may fail to exist generically depending on the utility level of the outside option. If they exist, they may induce the least desired outcome while, generically, asymmetric equilibria are uniquely determined by a positive maximal rate of attendance.

The sequential setting turns the static game into a dynamic economic law of the Cobweb type for which there always exist nontrivial $\epsilon-$perfect predictors implementing $\epsilon-$perfect steady states as stable outcomes. If zero participation is a Nash equilibrium of the game there exists a unique perfect predictor implementing the trivial equilibrium as a stable steady state. In general, Nash equilibria of the one-shot game are among the $\epsilon-$perfect foresight steady states of the dynamic model.

If agents randomize over indifferent decisions the induced random Cobweb law together with recursive predictors becomes an iterated function system (IFS). There exist unbiased predictors with associated stable stationary solutions for appropriate randomizations supporting nonzero asymmetric equilibria which are not mixed Nash equilibria of the one-shot game. However, the least desired outcome remains as the unique stable stationary outcome for $\epsilon = 0$ if it is a Nash equilibrium of the static game.

Keywords: El Farol, participation games, repeated play, forecasting, rational expectations, Cobweb models

JEL classification: C7, C73, D83, D84

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1 Introduction

The “El Farol problem” – or

going to a bar in Santa Fe every Friday night Arthur (1994).

– stands for a non-cooperative setting of $N$ agents/players who
– can participate in a joint activity where

the benefits to each individual depend on the number of participants
– or stay by themselves, i.e. chose an outside option

Related Scenarios

• participation in clubs, unions, cartels, political parties, location
• entry-exit decisions for firms in markets,
• mechanisms for supporting a local public project,
• segregation games (Schelling, 1969, 1971; Radi, Gardini & Avrutin, 2014)
Arthur (1994) argues that in a repeated or recursive setting, a stationary rational expectations solution may not exist since standard deductive reasoning by agents may not lead to a correct anticipation of the decision of others.

He shows by means of an example that adaptive expectations may induce non-stationary and erratic sequences of plays inconsistent with expectations.

This is taken as an indication for the need to search for solutions under bounded rationality.
This note shows that, generically

- there exists a unique trivial equilibrium of the El Farol game
- The sequential setting induces an economic law of the Cobweb type
- with unique perfect predictor and stable steady state.
- a randomization by agents to break indifferences converts the random Cobweb law into an iterated function system (IFS) with
  - stable stationary outcomes under rational expectations, inducing
  - plays with i.i.d. mixing strategies of the static game which
  - are not equilibria of the mixed extension of the static game

⇒ the apparent failure of agents to find stabilizing rational prediction rules, is not due to a non-existence of perfect rules, but rather

⇒ to their inability to identify the correct class of predictors from which the perfect ones can be chosen.
2 The El Farol Game

- Let $I = \{1, \ldots, i, \ldots, N\}$ denote the set of players/visitors to El Farol
- where $N \in \mathbb{N}$ is a relatively large number
- let $n \in [0, 1]$ denote the proportion of players present at El Farol.
- assume identical preferences for all players defined by a function $u : [0, 1] \times \{0, 1\} \rightarrow \mathbb{R}$

\[
(2.1) \quad u(n, x) = xv(n) + (1 - x)B \quad B > 0
\]

- where $v : [0, 1] \rightarrow [0, 1]$ is a concave unimodal function
- satisfying $v(0) = v(1) = 0$ and a unique positive interior maximum, i.e.
- for some $0 < \bar{n} < 1$, $v(\bar{n}) \geq v(n)$, for all $n \in (0, 1)$ with $v(\bar{n}) > v(0) = v(1)$.

agents prefer medium crowded bars to under- or overcrowded ones.
2.1 The Game in Normal Form

Denote by \( x := (x_1, \ldots, x_N) \) a list of choices for each player and define by

\[
\nu(x) := \frac{\sum_{j=1}^{N} x_j}{N}
\]

the proportion of players at El Farol induced by \( x \).

This yields a payoff function for each \( i \) given by \( \pi_i(x) := u(\nu(x), x_i) \).

The pair \( G := (I, \Pi) \) is a symmetric normal form game with payoff function

\[
\Pi(x) := \prod_{i=1}^{N} \pi_i(x) = \prod_{i=1}^{N} u(\nu(x), x_i).
\]

\( x = (1, \ldots, 1) =: 1 \) and \( x = (0, \ldots, 0) =: 0 \)

are the only candidates for symmetric Nash equilibria

since partial attendance is never symmetric.
Symmetric nontrivial equilibria are rare

- \( x = (1, \ldots, 1) \) is not a Nash equilibrium if \( B > v(1) \), for any concave \( v \).
- Conversely, if the condition \( B < v(1/N) \) holds, \( G \) has no nontrivial symmetric equilibrium in pure strategies.
- Clearly, if \( B > 0 \) and \( v(0) = v(1) = 0 \), then \( x = (1, \ldots, 1) \) is not a Nash equilibrium.
- Therefore, if the condition \( B > v(1/N) \) holds,
  - \( x = (0, \ldots, 0) = 0 \) is the unique symmetric Nash equilibrium for \( G \) in pure strategies
  - \( G \) has no nontrivial symmetric equilibrium in pure strategies.
- Of course, there is at least one mixed Nash equilibrium.
-\( \implies \) The symmetric Nash equilibrium induces the least desired outcome.
2.1 The Game in Normal Form

- Define \( v^+(B) := \{ n \in (0, 1) | v(n) \geq B \} \), the upper contour set of \( v \)

**Lemma 1.**

1. \( x = 0 \) is the unique symmetric Nash equilibrium of \( G \) if and only if \( B \geq v(1/N) \).

2. Let \( v^+(B) \cap \{1/N, \ldots, (N - 1)/N\} \neq \emptyset \).
   1. There exists a unique maximal group size \( \hat{k} \in \{1, \ldots, N - 1\} \) and associated list \( \hat{x} \in \{0, 1\}^N \) with \( \sum_{j=1}^{N} \hat{x}_j = \hat{k} \) such that \( 0 \neq \hat{x} \neq 1 \) is an asymmetric Nash equilibrium. \( \hat{k} \) is the only group size supporting asymmetric Nash equilibria if \( v(\hat{k}/N) > B \).
   2. If \( v(\hat{k}/N) = B \) and \( (\hat{k} - 1)/N \in v^+(B) \), then \( \hat{k} - 1 = \sum_{j=1}^{N} x_j \) is an additional group size with associated asymmetric Nash equilibrium \( x \).
   3. If \( v((\hat{k} - 1)/N) = v(\hat{k}/N) = B \), then all \( k/N \in v^+(B) \) support asymmetric Nash Equilibria.
Result:

- for ’generic’ concave unimodal utility functions and stand alone values $B$
- zero is the unique symmetric Nash equilibrium;
- asymmetric equilibria are defined by
- a unique participation number $0 < \hat{k} < N$;
- non zero equilibria induce a coordination problem
- results extend to heterogeneous preferences and values
Going to El Farol repeatedly

Assume that all players decide repeatedly whether to go to El Farol or not,
on the basis of a subjective belief, i.e. after making a forecast of
the relative number of players to be present.

The delay between the forecast and
the actual attendance given a forecast
induces a mapping from the space of forecasts to outcomes/attendance

Formally, let $0 \leq n^e \leq 1$ denote i’s forecast, which induces
the best response attendance map of each player

$$\xi(n^e) := \arg \max_{x \in \{0,1\}} u(n^e, x)$$

if he assumes that $n^e$ visitors (including himself) are there.
Lemma 2.
Let $v$ be continuous and concave such that there exists $n \in [0, 1]$ with $v(n) > B > v(1)$.

For $B < v(0)$, the best response correspondence $\xi$ has the form

$$\xi(n^e) := \begin{cases} 
1 & 0 \leq n^e < \bar{n}^e \\
\{0, 1\} & n^e = \bar{n}^e \\
0 & \bar{n}^e < n^e \leq 1
\end{cases}$$

where $\bar{n}^e$ satisfies $v(\bar{n}^e) = B$. 
If \( B > v(0) \),
there exist two positive numbers \( 0 < n_1^e < n_2^e < 1 \) defined by

\[
v(n_1^e) = v(n_2^e) = B
\]

such that

\[
\xi(n^e) := \begin{cases} 
0 & 0 \leq n^e < n_1^e \\
\{0, 1\} & n^e = n_1^e \\
1 & n_1^e < n^e < n_2^e \\
\{0, 1\} & n^e = n_2^e \\
0 & n_2^e < n^e \leq 1
\end{cases}
\]

The correspondence \( \xi : [0, 1] \rightarrow \{0, 1\} \) has a closed graph
(is upper hemi-continuous).
Definition 1. 
A pair of \((x, n^e) \in \{0, 1\}^N \times [0, 1]\) is called an equilibrium under best response with consistent homogeneous expectations if 

\[(3.4) \quad n^e = \nu(x) \quad \text{and} \quad x_i \in \xi(n^e) \quad \text{for every} \quad i \in I.\]

Equivalently, \(x = (x_1, \ldots, x_N)\) is an equilibrium if 

\[(3.5) \quad x_i \in \xi(\nu(x)) \quad \text{for every} \quad i \in I.\]

Result: 
– \(x = 0\) is the unique symmetric equilibrium 
– there may exist asymmetric equilibria 
– these are not Nash equilibria of the game \((I, \Pi)\) 
– they are “rare” in the space of \(\{\nu : [0, 1] \rightarrow [0, 1]; B > 0\}\)
3.1 The Dynamics of Expectations and Attendance under Repeated Play

Definition 2.

Let \( \Xi(n^e) := (\xi(n^e), \ldots, \xi(n^e)) \) denote the \( N \)-fold product of the best response correspondence \( \xi \). There exists a correspondence \( F : [0, 1] \rightarrow [0, 1] \) describing the outcome/attendance map \( F := \nu \circ \Xi \) defined as

\[
F(n^e) := \begin{cases} 
0 & 0 \leq n^e < n_1^e \\
\frac{1}{N}\{0, 1, 2, \ldots, N\} & n^e = n_1^e \\
1 & n_1^e < n^e < n_2^e \\
\frac{1}{N}\{0, 1, 2, \ldots, N\} & n^e = n_2^e \\
0 & n_2^e < n^e \leq 1
\end{cases}
\]

(3.6)

- Given the best response behavior of all agents for any forecast \( n^e \),
- an \( x \in \Xi(n^e) \) defines the induced next state of attendance \( n = \nu(x) \).
Sequential interpretation: let
\[-n_{t-1,t}^e\] denote the expected attendance at the end of date \(t-1\) for period \(t\)

\(\implies\) the mapping \(F\) describes possible next periods attendances

\[n_t \in F(n_{t-1,t}^e)\] under best response \(\implies\) call \(F\) the El Farol Law.

Properties: \(F\) is of the Cobweb type\(^1\).

\(\implies\) The dynamics of Cobweb models are driven exclusively by forecasts

\(\implies\) constant forecasts induce constant orbits

\(\implies\) constant forecasts are the only forecast rules which induce perfect foresight/rational expectations

\(\implies\) failure of convergence or of rationality (perfect foresight) means permanent systematic forecasting errors

\(^1\)\(F\) is of the Cobweb type since its form corresponds to the common Cobweb model of dynamic price theory See Ezekiel (1938); Waugh (1964); Nerlove (1958); Pashigian (1987); Böhm & Wenzelburger (1999)
3.2 Predictors and Dynamics

– Forecasting mechanisms or rules are called predictors
– which are mappings from past data to forecasts
– Stationary (recursive) predictors induce an autonomous dynamical system.
– Let $T \geq 1$ denote the length of a finite history of observations

$$(x_{-T}, n_{-T}, n^e_{-T}) := (x_\tau, n_\tau, n^e_\tau)^{-1}_{T=1} \in \{0, 1\}^{TN} \times [0, 1]^{2T}$$

where $n^e_\tau \equiv n^e_{\tau-1, \tau}$ denotes the forecast made in period $\tau - 1$ for $\tau$.

**Definition 3.** A predictor is a mapping $\psi : \{0, 1\}^{TN} \times [0, 1]^{2T} \rightarrow [0, 1],$

$$(x_{-T}, n_{-T}, n^e_{-T}) \mapsto \psi (x_{-T}, n_{-T}, n^e_{-T}) = n^e.$$ 

**Definition 4.** Given the El Farol Law $F$ and a predictor $\psi$,

the pair of mappings $(\psi, F \circ \psi)$ defines the one-step movement of a set valued dynamical system on $\{0, 1\}^{TN} \times [0, 1]^{2T}$. 

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An orbit of the system is a sequence \( \{(x_t, n_t, n_t^e)\}_{t=0}^{\infty} \) such that

\[
\begin{align*}
    n_t^e &= \psi(x_{t-T}, n_{t-T}, n_{t-T}^e) \\
    n_t &\in F(\psi(x_{t-T}, n_{t-T}, n_{t-T}^e))
\end{align*}
\]

(3.7)

with \( n_t = \nu(x_t) \) and \( x_t \in \Xi(n_t^e) \).

- The notion of a perfect predictor is defined in two steps
- using the methodology proposed in Böhm & Wenzelburger (1999).
Definition 5. An orbit \( \{(x_t, n_t, n_t^e)\}_{t=0}^{\infty} \) of \((\psi, F \circ \psi)\) is said to be \(\epsilon\)-perfect for \(\epsilon \geq 0\) if

\[
|n_t^e - n_t| \leq \epsilon \quad \text{for all} \quad t \geq 0.
\]

It is said to have the perfect foresight property if \(\epsilon = 0\), or equivalently if

\[
n_t = n_t^e
\]

holds for all \(t\) with \(n_t^e = \psi(x_{t-T}, n_{t-T}, n_{t-T}^e)\), \(x_t \in \Xi(n_t^e)\) and \(n_t = \nu(x_t)\).
Definition 6. A predictor $\psi^*$ is called $\epsilon$-perfect for $\epsilon \geq 0$ if it induces an $\epsilon$-perfect foresight orbit. It is called perfect for $\epsilon = 0$. Therefore, $\psi^*$ is a perfect predictor if

$$n_{t+1} = n^e_{t+1} = \psi^*(x_t, n_t, n^e_t) \quad \text{for all} \quad t \geq 0,$$

or equivalently if

$$\psi^*(x_t, n_t, n^e_t) \in F(\psi^*(x_t, n_t, n^e_t))$$

holds for all $t$ and $n^e_{t+1} = \psi(x_t, n_t, n^e_t)$, $x_t \in \Xi(n^e_t)$ and $n_t = \nu(x_t)$. 
One obtains the following result.

**Proposition 1.**

Assume $B > v(0)$ and let zero be the unique fixed point of the El Farol Law $F$ as given in Definition 2.

1. The constant predictor $\psi^*(x_{-T}, n_{-T}, n_{e-T}) \equiv 0$ is the unique globally perfect predictor for $F$.

2. The state $x = n = n_e = 0$ is the unique globally stable steady state of the system $(\psi^*, F \circ \psi^*)$.

3. There exist $\epsilon_i > 0$, $i = 1, 2$, such that the constant predictors $\psi_i(x_{-T}, n_{-T}, n_{e-T}) \equiv n_i^e$ are $\epsilon_i$-perfect for $i = 1, 2$ respectively, i.e. there exist $(x_i, n_i) = (x_i, \nu(x_i))$ such that

$$|n_i - n_i^e| = |\nu(x_i) - n_i^e| \leq \epsilon_i, \quad i = 1, 2$$

4. $(x_i, n_i, n_i^e)$ are globally stable for $(\psi_i, F \circ \psi_i)$, $i = 1, 2$ respectively.
Implications:

- The El Farol Law indeed has a clear unique globally stable solution
- obtainable as the best response outcome under
  full rationality and noncooperative behavior of all agents
- provided a rational outside analyst suggests that every agent chooses the
  perfect predictor.
- In such case, convergence under perfect foresight is obtained and no cycles
  occur.
- The uniqueness of the perfect predictor (in the space of unbiased recursive
  forecasting rules) implies that
- no other stationary predictor could induce a limiting self confirming orbit.
- There is no room or need for considerations of bounded rationality.
3.3 A Stochastic El Farol Law

When agents randomize – mixing of best responses

– Let the forecast $n^e$ be interpreted as the subjective prediction of each agent for the mean attendance next period.

To break indifferences from (3.3), define for every agent $i \in N$ a pair of best response functions $\xi_{max} : [0, 1] \rightarrow \{0, 1\}$ and $\xi_{min} : [0, 1] \rightarrow \{0, 1\}$

\[
\xi_{max}(n^e) := \arg \max\{x | x \in \xi(n^e)\} \quad \text{and} \quad \xi_{min}(n^e) := \arg \min\{x | x \in \xi(n^e)\}
\]

– which satisfy $\xi_{max}(n^e) = \xi_{min}(n^e)$ for $n^e \neq n^e_1, n^e_2$.

– Let $P := (p_1, \ldots, p_N)$, $0 < p_i < 1$ denote a list of probability distributions on $\{0, 1\}$, one for each $i$

– according to which $i$ randomizes over the pair $(\xi_{min}(n^e), \xi_{max}(n^e))$ of best response functions.
For a given realization $w \in \{0, 1\}^N$ define the random choice of $i$ as
\begin{equation}
\xi(w_i, n^e) := w_i \xi_{\text{max}}(n^e) + (1 - w_i) \xi_{\text{min}}(n^e).
\end{equation}

\implies a random family of mappings $F(\cdot, n^e) : [0, 1] \to [0, 1]$ given by
\begin{equation}
F(w_1, \ldots, w_n, n^e) := \frac{1}{N} \sum \xi_i(w_i, n^e)
\end{equation}

- describing the one step random change of the attendance as
- a stochastic difference equation
\begin{equation}
n_t = F(w_t, n_t^e).
\end{equation}

- Thus, $F(w, \cdot)$ is a random El Farol Law of the Cobweb type.
Rational Expectations

- Let \( n^e = \psi(x_{-T}, n_{-T}, n^e_{-T}) \) denote a mean value predictor for \( F \). Then,

\[
(3.15) \quad n^e = \psi(x_{-T}, n_{-T}, n^e_{-T}) \quad n = F(w, \psi(x_{-T}, n_{-T}, n^e_{-T}))
\]

- defines the random one step evolution of expectations and attendance.
- Therefore, given the probability measure \( P \) on \( \{0, 1\}^N \),
- the list \( \{(\psi, F(w, \psi); P)\} \) becomes an **iterated function system IFS**\(^2\)
- which induces random orbits \( \{(n^e_{-T}, n_{-T})\}^\infty_0 \) for any sample path
  \[ \omega := (\ldots, w_0, w_1, \ldots, w_{\tau}, \ldots) \in \Omega := \{\{0, 1\}^N\}^N \] and \( (n^e_0, n_0) \).
- The performance of a mean predictor is measured by the
  mean of the prediction error

\[
(3.16) \quad \mathbb{E} \{n^e_t - n_t\} = n^e_t - \mathbb{E} \{F(w_t, \psi(x_{-T}, n_{-T}, n^e_{-T}))\}.
\]

\(^2\) (see Barnsley, 1988; Arnold & Crauel, 1992)
Definition 7. A mean value predictor $\psi^*$ is called \textit{unbiased} if its induced mean error is zero, i.e. if

\begin{equation}
\mathbb{E}_P \{ F(w_t, \psi^*(x_{-T}, n_{-T}, n^e_{-T})) \} = \psi^*(x_{-T}, n_{-T}, n^e_{-T})
\end{equation}

Lemma 3.

Let $F$ denote the random El Farol Law defined in (3.13) and denote by

$$(EF)(n^e, P) := \mathbb{E}_P \{ F(\cdot, n^e) \}$$

its expected value function with respect to $P$.

- $\psi^*$ is an unbiased predictor for $F$,
- if $\psi^*$ predicts a fixed point of the mean El Farol Law $(EF)$.

- By construction the mean law is discontinuous at $n^e_1$ and $n^e_2$ such that

\footnote{proceed as in Böhm & Wenzelburger (2002)}
\begin{equation}
(3.18) \quad (EF)(n^e, P) := \begin{cases}
0 & 0 \leq n^e < n_1^e \\
\frac{1}{N} \sum_{j=1}^{N} p_j & n^e = n_1^e \\
1 & n_1^e < n^e < n_2^e \\
\frac{1}{N} \sum_{j=1}^{N} p_j & n^e = n_2^e \\
0 & n_2^e < n^e \leq 1
\end{cases}
\end{equation}

– and generically $n_1^e \neq (EF)(n_1^e, P)$ and $n_2^e \neq (EF)(n_2^e, P)$,
– making zero the unique fixed point of the mean law
– implying the following result.
**Theorem 1.** Let $F$ denote a random law for $(v, B)$ with $P$ given.

(i) If $B > v(0)$:

1. $\psi^* \equiv 0$ is the unique unbiased predictor if and only if
   \[ \sum_{1}^{N} p_j \neq N \n_i^e, \ i = 1, 2. \]

2. Zero is the unique stable solution of \[ \{(\psi^*, F(w, \psi^*)); P \} \).

3. There exist two unbiased predictors with constant value $\psi_i \equiv n_i^e$, $i=1,2$, if and only if
   \[ \sum_{j} p_i^j = N \n_i^e, \ P_i := (p_i^1, \ldots, p_i^j, \ldots, p_i^N), \]

4. All random orbits \( \{n_i^e, n_t\}_{t=0}^{\infty} \) of the IFS \( \{(\psi_i, F(w, \psi_i); P_i\} \) are stationary solutions with sample mean $n_i^e$ and rational expectations $\mathbb{E}_{P_i}(n_t) = n_i^e, \ i = 1, 2$.

(ii) If $B < v(0)$:

1. The constant mean value predictor $\bar{n}^e \equiv \bar{\psi}$ is the unique unbiased predictor for $(v, B, P)$ if and only if $\sum \bar{p}_j = N \bar{n}^e$.

2. All random orbits \( \{ (\bar{n}^e, n_t) \}_{t=0}^{\infty} \) of the iterated function system \( \{(\bar{\psi}, F(w, \bar{\psi})); \bar{P}\} \) are stationary solutions with sample mean $\bar{n}^e$ and rational expectations $\mathbb{E}_{\bar{P}}(n_t) = \bar{n}^e$. 

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Corollary 1.

There exist convex sets $P^i \subset [0, 1]^N$ of probability distributions $P^i = (p^i_1, \ldots, p^i_N) \in P^i, i = 1, 2$ such that

1. the constant mean predictors $\psi^i = n^e_i$ is unbiased for $P^i \in P^i, i = 1, 2$,
2. $n^e_i$ is the unique positive fixed point of the mean law $n^e_i = (EF)(n^e_i, P^i)$
3. All random orbits $\{(n^e_i, n_{r})\}$ of the IFS defined by $\{\psi^i, F(w) \circ \psi^i, P^i\}$ converge to $(n^e_i, n^e_i)$, $i = 1, 2$ in sample mean.
Summary of Results:

- there is no non-existence issue under rationality with random mixing

⇒ the least desirable attendance remains a stable outcome under unbiased predictions and any i.i.d. randomization of best responses,

- the two positive critical levels $0 < n_1^e < n_2^e < 1$ are fixed points of the mean law for specific probability distributions

- any associated orbit $\{x_\tau = ((\xi(w_j^\tau, n_i^e))_{j=1}^N)\}_{\tau=0}^\infty$ is a sample path of

- plays of a mixed strategy with probabilities $P^i = (p_1^i, \ldots, p_N^i)$, $i = 1, 2$.

⇒ positive stationary sequences of attendance with rational expectations require specific forecasts and specific randomizations

it is unclear which of these probabilities are Nash equilibria of the mixed extension of the El Farol game.
4 Conclusions – Implications – Extensions

4.1 Existence and the role of the essential discontinuity
– there is no general non-existence issue under rationality
  in El Farol type problems

However the least desired outcome is almost always a best response state
– when outside options enter as a no participation outcome
⇒ optimizing with an outside option creates an
  unavoidable nonconvexity
  which is the true source for the implications.

⇒ The non-existence of nontrivial or active stationary states is
  caused by the structural discontinuity of the best-response map
– arising from a nonconcave maximization problem.
4.2 Heterogeneity of agents

– None of the above results loose their general validity
– when agents’ preferences and outside options are heterogeneous.
– These imply a discrete dispersion of finitely many discontinuities
– an increase of possible nontrivial stationary states,
– but no smoothing implications or continuity
– leaving the zero attendance state always as a stable configuration
  of an El Farol type Law with rational expectations.
– It is well known and part of common economic reasoning in markets that
– heterogeneous expectations often cause nontrivial and nonstationary orbits,
– but the heterogeneity of forecasts cannot be self-confirming for all agents.
References


